Book Three
Math

Introduction

This book will take you through three sections regarding SAT math: Simplifying Structure, Success in Strategies, and Focusing on Formulas. This order is intentional.

Simplifying Structure breaks down every aspect of math that you will encounter on the SAT including: The different levels of math, the different kinds of math, all concepts to review, and the number of problems to expect on the math portion. This system assumes that you have taken and understood the concepts discussed in the following classes: pre-algebra, algebra 1, algebra 2, and trigonometry. This course teaches no subject matter regarding specific concepts, it simply shows you which subjects to review and how to apply your knowledge of these concepts specifically to the SAT.

The second section will be "Success in Strategies" which gives an in-depth breakdown of how strategies can be applied to optimize time. These strategies will cover how to approach problems, how to think about problems, how to manage time, and more.

The third section will be "Focusing on Formulas". Recognizing which formula to use on the SAT will save you time and effort. This section aims to properly supply you with all the different formulas for the test to prepare you in the best way possible.

Simplifying Structure

Time and text

The math section includes two different sections: the "No-Calculator" section and the "Calculator OK" section.

The no calculator section consists of sixteen multiple choice questions and four grid-in questions. You have 25 minutes to complete the all the answers and fill in the scantron, which allows only around a minute for each problem and a couple minutes to fill out the scantron.

The Calculator OK section has thirty-two multiple choice questions and six grid-in questions. You are allowed 55 minutes to complete this section which approximates to roughly a minute and a half per problem.

Types of questions

Inside the two math sections are four hidden categories: Heart of Algebra, Problem Solving and Data Analysis, Passport to Advanced Mathematics, and Additional topics. These are the general categories that describe the content that is in the SAT. Understanding the difference between these sections will better equip you to evaluate where you are at and what you need to improve. Pay close attention to the content described in the following paragraphs because it will help you understand EXACTLY what to expect from the math sections.

Heart of Algebra focuses mainly on concepts discussed in pre-algebra and algebra 1 but seems to put the most emphasis on the following concepts: Linear equations and systems of linear equations. A linear equation is an equation that can be graphed, and systems of linear equations simply means two or more linear equations in the same problem. These problems often appear in the form of a word problem that you must convert into an equation, or simply an equation. There will be 19 different types of problems that appear on the test (33% of the math test) that cover "The Heart of Algebra" so it is important that you understand these concepts. If you would like a more in-depth breakdown of specific problem types in the Heart of Algebra, and a short strategy, view the Focus Forms in Book Five.

Problem Solving and Data Analysis covers topics including percentages, units, and probabilities. Unlike most of the problems in the heart of algebra, this section does not only incorporate concepts, but also tests students on their logical thinking. The SAT includes 17 different questions from this category which makes up 29% of the math test.

Passport to Advanced Mathematics includes questions discussed in algebra 2. This section encompasses most of the problem areas for students. The focus of this section is: quadratics, nonlinear equations and non-linear graphs. Memorizing formulas for this

section is essential. Many of the questions simply are matching formulas to an image, or rearranging formulas to be in the correct format. Additionally, it is important to practice these questions to recognize what the SAT wants you to do. There are 16 questions from the Passport to Advanced Mathematics that will be on the math section which makes up 27% of test. When solving these problems, you must combine all your skills: logical thinking, understanding concepts, and memorizing formulas. Each is a skill to learn. If you are struggling with any portion of the math section, I would suggest learning the concept first, then learning how to incorporate formulas into problems, and lastly combining logical thinking with the strategies listed below for optimal results.

Additional Topics is the smallest section of the SAT. The SAT only covers the topics in this section through six questions in the test (10%). The concepts are the following: area/volume problems, trigonometry, complex numbers, and circle equations. These questions often occur as grid-in questions. This section is formula heavy. Throughout the test you should anticipate at least 1 circle equation problem, 2-3 area or volume problems, 2 trigonometry problems (90% of the time they will be right triangle problems), any sometimes you will encounter a complex number (i) problem. These problems are straightforward; however, they can be more difficult because they are often found at the end of the sections.

Success in Strategies

Always read the question

After you have finished working on the problem, always go back and read what the question was asking. Often students will get halfway through solving the problem or solve for *x* when the question required them to solve for 3*x*.

Plug in the Answer

Working on a problem to get an answer and then simply selecting the answer you got from the problem disables a huge advantage the multiple-choice question often gives you. Often with long algebra problems you might benefit from plugging in the multiple-choice answers and working the problem backwards. Additionally, it can never hurt to refer to the answers in the process of solving the problem.

Process of Elimination

Using the process of elimination is a combination of common sense and logical thinking. For example:

$[21(-31) + 41(2)]$ Simplify this equation.

    A. -569
    B. 569
    C. -63
    D. 63

We can tell just by looking at the equation that the answer will have to be negative, therefore eliminating options B and D. Then by using common sense and estimating, we can see that the number will clearly be in the negative hundreds leaving only option A. This reduces the time from a problem that takes nearly a minute to write out and work, to a problem that takes roughly 5-10 seconds to work. When you encounter easy problems, applying this strategy will allow you to have more time to work on the harder problems in the test and conserve brain power.

You can also use this concept in more complicated problems, although I suggest eliminating answers as you go, instead of working through the entire problem in your head.

If you can, eliminate obviously wrong answers right off the bat, and then only work the problem until you have eliminated the other wrong answers. It is common that you will be able to eliminate three answers before finishing the problem.

Take Problems One Step at a Time

Often problems in the SAT (specifically long word problems) can be very difficult if you try to understand the whole thing at once. Take it SLOW. Read one sentence at a time and

digest what it is saying before you move onto the next one. Writing down what parts of sentences mean can be very beneficial especially if you struggle with understanding the problem.

Do not waste time on your calculator if you do not need to

Time is the biggest factor in the SAT so do not waste time using your calculator unnecessarily.

Pacing

If you spend more than one minute on a problem, mark it and come back to it. If you are struggling with a problem and you spend a lot of time on it, there is still a chance you can get it wrong, AND you will be more pressured for time with a problem you could get right if you had the time to properly work it.

Scenario 1: You are on the No calculator Math section, problem 4. You struggle and spend five minutes trying to figure it out. You think you got the answer, but you are not confident. Now you wasted four minutes and threw off your testing confidence because now you feel like you don't know what you're doing. You get to problem 19 and you have 30 seconds left to do 2 problems. If you had not wasted time on problem 4 you would be fine but now you are stressed and rushed. So, you either guess, or you do not finish the problem. So instead of just getting problem 4 wrong, you got problem 4, 19, and 20 wrong.

Scenario 2: You see that you are struggling with problem 4, so you mark it and move on with your test with the same confidence you had before. Since you spend only 1 minute or less on each problem, you finish all the questions that you know FOR SURE how to work and get them all correct. Now, you have 5 additional minutes to go back and look at the questions you were struggling with. Instead of getting 3 questions incorrect you maybe got one and you were much less stressed during the process.

Focusing on Formulas

Preface

There are two main times when formulas are used in the SAT: Plugging in formulas and formula recognition. Word problems might want you to convert words into a specific formula, you might have to rearrange a formula into the correct format. There are many examples of when you need formulas, but I will spare you the lecture. What is important is that you can recognize what formulas you need to use at what time. To understand which formulas, you are supposed to use, you must know which formulas are available!

Below is a list of formulas that are necessary to memorize (and some rules too) to maximize your familiarity with the test. Many of these formulas are used across several portions of the test, but I have tried my best to sort them where they are the most applicable.

Summary of Different Formulas

Heart of Algebra:

Standard formula (y-intercept)

- $Ax + By = C$

Factored formula (x-intercept)

- $a^2 - 2ab + b$

Problem Solving and Data Analysis

This section does not focus on specific formulas. Rather, it references properly reading formulas. You need to understand the different parts of exponential and linear formulas and what the role of each part is.

Mean/average:

- $\dfrac{\text{sum of terms}}{\text{number of terms.}}$

Speed:

- $\dfrac{\text{distance}}{\text{total time}}$

Passport to Advanced Mathematics

Quadratic formula

- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant formula

- $b^2 - 4ac$

Vertex formula

- $y = a(x - h)^2 + k$

Difference of cubes

- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

Sum of cubes

- $a^3 + b^3 = (a + b)(a^2 - b^2 + ab)$

Difference of squares

- $a^2 - b^2 = (a - b)(a + b)$

Slope intercept formula

- $y = mx + b$

Parabola equation

- $y = a(x - p)(x - q)$

Rules for exponents:

- $a^2(a^2) = a^{2+2}$

- $\dfrac{a^5}{a^3} = a^{5-3}$

- $(a^2)^2 = a^{2 \times 2}$

- $3^4 \times 3^5 = 3^9$

- $6^5 \times 4^5 = (6 + 4)^5$

- $5^5 \times 4^3 \neq (5 + 4)^{5+3}$

- $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

- $\sqrt[2]{\dfrac{a}{b}} = \dfrac{\sqrt[2]{a}}{\sqrt[2]{b}}$

- $\sqrt[2]{ab} = \sqrt[2]{a}\sqrt[2]{b}$

- $x^{-n} = \dfrac{1}{x^n}$

Additional Topics

Circle formula

- $(x - h)^2 + (y - k)^2 = r^2$
- Think: (point – center) + (point – center) = Radius

Pythagorean theorem

- $a^2 + b^2 = c^2$

Sin cos tan rules

SOH: $SINE = \frac{opposite}{Hypotenuse}$

CAH: $COS = \frac{Adjacent}{Hypotenuse}$

TAH: $TAN = \frac{Adjacent}{Hypotenuse}$

Length of the arc

- $s = r\theta$

Area of the sector

- $A = \frac{1}{2}\theta r^2$

Laws:

- There are 360 degrees in a circle.
- 2π = radians (θ)
- Triangles have 180 degrees of angles.

Formulas Shown on Test

Circumference of a circle

Area of a circle

Area of a rectangle

Area of a triangle

Properties of a special right triangle (isosceles triangles)

Properties of a special right triangle (30, 60, 90 Degree triangle)

Volume of a rectangular Solid

Volume of a cylinder

Volume of a sphere

Volume of a cone

Volume of a pyramid

Summary

-   Introduction
-   Time and Text
-   Type of Questions
-   Six "Strategies for Success"
-   Forty-three different formulas to memorize